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PROGRESS REPORT

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Robert E. Roberson

Principal Investigator

RESEARCH BY THE PRINCIPAL INVESTIGATOR

During the second and third quarters of the 1966-67 academic year the Principal Investigator was on sabbatical leave from UCLA. Accordingly, his studies in this problem area were conducted partly in Paris, where he resided during most of the spring of 1967. Although Paris is a charming city, the weather is abominable, and the site was chosen only because of access to the excellent library facilities there. Most of his time during this period was spent in study and writing: the libraries involved were mainly the Bibliothèque Nationale, the mathematics library of the École Normale, the general library of the École Nationale Superieure de l'Aeronautique, and a few small specialized libraries.

He feels that he made very substantial progress during this time in developing a coherently organized treatment of rotational dynamics as regards both the classical sense of this subject and its modern counterparts in the application area of spacecraft rotation. One task accomplished was to bring within a single framework the various four-parameter methods for representing rotation. The unity of the subject is not novel in a purely mathematical sense, but it is not generally recognized by engineers concerned with the dynamic modeling of spacecraft rotation. (A tutorial paper on this is planned, but so far exists only in a more expanded written version that would not be suitable for journal publication.) He also has given a new and considerably improved derivation of the kinematical equations for a three-parameter algebraic representation of rotation -- the Euler-Rodrigues parameters. A short paper on this is complete: "Kinematical Equations for Bodies whose Rotation is Described by the Euler-Rodrigues Parameters" (5 copies appended hereto) which will be submitted for publication shortly. Also in the area of kinematics, he has developed, with Professor P. W. Likins, a convenient matrix linearization technique which can be used to shorten the linearization process significantly. This has been taught to students, beginning in March, but is not yet in publishable form.

The Principal Investigator also has devoted some time to improving his multi-body Eulerian dynamical formalisms, but does not feel this is quite ripe for publication. Further work in this area is planned during the coming 6-month period.

As regards spacecraft problems per se, he has extended the work reported in the previous progress report (his IAF paper described on p. 1 thereof) so that some stability results now have been obtained. This work is continuing, but only at a low level. He has written a short paper entitled "Torque on a Satellite in a General Gravitational Field" (5 copies appended hereto) which will shortly be submitted for publication. This work gives an alternative derivation of some known results in a much more compact and straightforward form, leading to a matrix form which is very convenient for numerical simulation. However, the derivation also generalizes previously available results in several particulars: i. e., a more general reference frame for spacecraft orientation is used, no assumption is required about the use of principal axes of inertia for body axes or the use of any specific parameterization of the direction cosine matrix. Moreover, the field of the nearby body is not specialized (except as an illustration) to an inverse square field with a single oblateness term added. More general potentials can be handled straightforwardly, and this may prove valuable in the case of lunar satellites when one wishes to investigate such effects as that of lunar triaxiality on altitude stability.

Despite his best intentions, the Principal Investigator has yet to make any substantial progress on the dynamics of orbiting flexible bodies. He has begun some reading that bears on the problem, but there have been too many other interesting problems to claim his attention.

RESEARCH BY GRADUATE STUDENT,
ANDREW H. MILSTEAD

An investigation was initiated to study the motion of a mass ring in a gravitational field. Initial assumptions were an inertially fixed point gravitational source and a rigid circular ring. Integrations were performed over the ring to determine its gravitational potential, and this potential was then approximated for ring radius ρ much less than and much greater than Earth radius R. Force and torque components were derived and the complete motion of the ring was expressed in terms of six second order nonlinear differential equations. Some stability properties were investigated for certain special cases.

For $R >> \rho$ it was found that if the ring's a titude could be controlled in a certain manner by some external agent, the force component perpendicular to R could cause a change of orbit inclination. For the case of $R << \rho$, the translational-rotational coupling is of the same order as the radial force. It was found that when the gravitational source is restricted to the plane of the ring a divergent solution for the motion of the ring's mass center results (i. e., R tends to increase without bound until the "small R" assumption is violated – the manner in which this divergence occurs is a function of initial conditions, and the equations for the resulting asymptotic trajectory have been solved.)* One non-divergent solution to the translational and rotational equations was found. This solution corresponds to a circular path for the rings center of mass (i. e., R is constant) and with the ring's axis colinear with R. Linearization of the equations about this solution shows that the solution is unstable in altitude and therefore, because of the strong rotation-translation coupling, is also unstable in the center of mass motion.

*This solution was found to be stable with respect to perturbations of the ring along its axis.

Plans for the next reporting period are to discontinue purely mathematical dynamical investigations, and to begin to explore other aspects of control feasibility for such ring satellites.